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(10) Maurice Mizrahi

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CENTER FOR NAVAL ANALYSES

2000 North Beauregard Street, Alexandria, Virginia 22311

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A TARGETING PROBLEM EXACT vs. EXPECTED-VALUE APPROACHES

Maurice Mizrahi

Naval Warfare Analysis Group

CENTER FOR NAVAL ANALYSES

2000 North Beauregard Street, Alexandria, Virginia 22311

ABSTRACT

This paper solves a targeting problem exactly and compares the exact solution with several reasonable expected-value approaches in order to gauge their accuracy. The scenario consists of air-to-surface missiles fired simultaneously at a ship formation defended with point defenses. The problem is to find the probability that a given ship will survive the raid. Each ship is present with a given probability and possesses a targeting "weight" determining the likelihood that it will be selected for targeting among the other ships present. The method can be extended to further layers of defense under certain conditions. Each solution is presented in 2 forms: a finite sum and a definite integral, each being easier to handle than the other under certain conditions. Examples are given. One important conclusion is the spectacular breakdown of reasonable expected-value approaches.

A TARGETING PROBLEM: EXACT VS. EXPECTED-VALUE APPROACHES

INTRODUCTION

The purpose of this paper is twofold: first, to solve a targeting problem exactly, and second, to compare the exact solution with several possible expected-value approximations in order to gauge their accuracy.

The targeting problem consists of a wave of bomber aircraft simultaneously firing missiles at a ship formation. Before the first bomber raid, all the ships are present, but after the first raid a ship is present only with a certain probability -- depending on whether it was sunk or not. Strictly speaking, these probabilities are not independent (a missile can hit only 1 ship: when it does, it is no longer available to hit another ship). However, if they are assumed independent, multiple raids can be analyzed sequentially with the method presented here. This method exactly calculates the probability that a given ship is present after a raid from the probabilities of ship presence before the raid and other relevant parameters (number of missiles launched, ship targeting weights, and ship vulnerability data).

The bombers typically face many layers of defenses, but the model described here considers only those missiles facing the last layer, namely the ships' point defenses, if any. Nevertheless, the method can be extended to further layers of ship defense -- such as SAM barriers or outer air battle of fighters vs. bombers.

Six expected-value approaches are presented as possible alternatives to the "exact" method. Their obvious advantage is that they are easy to use and require far less computer time and space. Two numerical examples help determine which, if any, of the 6 approximations are acceptable and under what conditions they are acceptable.

THE PROBLEM

The object is:

(1) To calculate the probability that a given ship in a formation is targeted by a given number of incoming antiship missiles (ASMs), and

(2) To calculate the probability that a given ship will survive the raid.

(We shall see that it is not always necessary to calculate (1) in order to obtain (2).) The inputs are:

n : maximum number of ships in formation

a_i : probability that ith ship is present in formation (the a_is are assumed independent)

w_i : weight of ith ship (a positive integer)

M : total number of ASMs launched (the missiles are considered independent)

$L_i(m)$: lethality function, i.e., probability that the ith ship is sunk if hit by m missiles (or, if m missiles attempt to penetrate its point defenses).

The constant weight assigned to each ship determines the probability that the ship will be targeted. For example, the probability that the 5th ship is targeted given that the 2nd, 5th, 7th, and 10th ships alone are present is

$$w_5/(w_2 + w_5 + w_7 + w_{10}) .$$

THE SOLUTION

All solutions will be expressed both as sums and integrals. In general, the larger the number of ships n the longer it takes to calculate the sum (2^{n-1} terms) and the larger the number of missiles M the longer it takes to calculate the integral (up to M terms in the integrand).

Targeting Probabilities

$P_i(m|M) \equiv$ probability that ith ship is targeted by exactly m missiles given that M missiles are launched and given that ship i is present

$$\equiv \sum_{k=1}^{n} \sum_{\{j_2,\ldots,j_k\} = \{1,\ldots,n;\text{except } i\}} a_{j_2} a_{j_3} \cdots a_{j_k}$$

$$\cdot (1-a_{j_{k+1}}) \ldots (1-a_{j_n}) \binom{M}{m} p_i^m (1-p_i)^{M-m} \quad . \tag{1}$$

where

$$p_i \equiv \frac{w_i}{w_i + w_{j_2} + \ldots + w_{j_k}} \quad . \tag{2}$$

This sum can be expressed as the following definite integral for $m \neq 0$:

$$P_i(m|M) = w_i \binom{M}{m} \int_0^1 f_i(x)\,dx \tag{3}$$

where

$$f_i(x) \equiv g_i(x) x^{w_i - 1} \prod_{j=1; \neq i}^{n} (a_j x^{w_j} + 1 - a_j) \tag{4}$$

$$g_i(x) \equiv \sum_{\ell=0}^{M-m} \binom{M-m}{\ell} (-1)^{\ell} \frac{(-w_i \text{Log} x)^{\ell+m-1}}{(\ell+m-1)!} \tag{5}$$

For $m=0$,

$$P_i(0|M) = 1 + w_i \int_0^1 dx\, h_i(x) x^{w_i - 1} \prod_{j=1; \neq i}^{n} (a_j x^{w_j} + 1 - a_j) \tag{6}$$

where

$$h_i(x) \equiv \sum_{\ell=1}^{M} \binom{M}{\ell} (-1)^{\ell} (-w_i \text{Log} x)^{\ell-1} / (\ell-1)! \tag{6a}$$

It is observed that the distribution is unimodal (rises, peaks, then falls), except for the fact that it may rise abruptly at the endpoint $m=M$. This corresponds to the configuration where ship i is the only one present, in which case it catches all the missiles.

Survival Probabilities

The probability that ship i will be present after the raid is:

$$a'_i = a_i [1 - P_{Ki}(M)] \, , \tag{7}$$

where

$P_{Ki}(M) \equiv$ probability that ship i is sunk given that it is present and that M missiles are launched on the formation

$$= \sum_{m=0}^{M} L_i(m) \, P_i(m|M) \, . \tag{8}$$

Note that, strictly speaking, this scheme cannot be iterated for several missile raids because the a'_i are not independent (as the a_i were assumed to be)[1].

Specific forms for the lethality function $L_i(m)$ often enable one to perform the above sum directly, without having to specifically calculate the targeting distribution $P_i(m|M)$. Consider the following form:

$$L_i(m) = 1 - (1-d_i)^m + c_i m(1-b_i)^m \quad , \tag{9}$$

where d_i, c_i, and b_i are free parameters determined by curve-fitting the lethality data (frequently, a simple power law, i.e., $c_i=0$, is sufficient.

Then

$$P_{Ki}(M) = \sum_{k=1}^{n} \sum_{\{j_2,\ldots,j_k\} = \{1,\ldots,n;\text{except } i\}} a_{j_2}\ldots a_{j_k}(1-a_{j_{k+1}})\ldots$$

$$(1-a_{j_n})\left[1 - (1-d_i p_i)^M + Mc_i p_i (1-b_i)(1-b_i p_i)^{M-1}\right] \tag{10}$$

with p_i given in equation (2). The integral form is

$$P_{Ki}(M) = w_i \int_0^1 dx\, u_i(x) x^{w_i-1} \prod_{j=1,\neq i}^{n} (a_j x^{w_j} + 1 - a_j) \tag{11}$$

---

[1] To see this, consider a formation of 2 ships with equal weights and $a_i=(1,1)$ targeted by 1 missile with lethality 1 (nuclear missile). Then $a'_i=(.5,.5)$. If the a'_i were independent, the probability that both ships would be sunk $[(1-.5)^2 = .25]$ would be nonzero, clearly an impossibility since the missile can only hit one ship. The results shown here are strictly correct only if the initial probabilities of presence (a_i) are truly independent. This means that the a_is cannot themselves be the result of a missile raid. They could be, for example, the result of independent attacks on each ship prior to the bombing raid.

where

$$u_i(x) \equiv \sum_{\ell=0}^{M-1} (M-\ell)\binom{M}{\ell}(-1)^{\ell} \frac{(-w_i \mathrm{Log}x)^{\ell}}{\ell!} \left[\frac{d_i^{\ell+1}}{\ell+1} + c_i(1-b_i)b_i^{\ell} \right] . \qquad (12)$$

For $c_i=0$, $u(x)$ simplifies to:

$$u_i(x) = -d_i \sum_{\ell=1}^{M} \binom{M}{\ell}(-1)^{\ell} \frac{(-w_i d_i \mathrm{Log}x)^{\ell-1}}{(\ell-1)!} . \qquad (13)$$

If the lethality data are too irregular to be fitted by (9), other formulas, with more free parameters, can be constructed which might enable one to carry out the sum in (8) exactly. These lethality formulas must satisfy $0 \leq L_i(m) < 1$, $L_i(0) = 0$, and $L_i(\infty) = 1$. An example is

$$L_i(m) = 1 - (1-d_i)^m + P_1(m)(1-b_i)^m + P_2(m)(1-b'_i)^m + \ldots,$$

where the $P_j(m)$ are polynominals in m with no constant term whose coefficients are free parameters depending on i.

Even when specific forms for the lethality function $L_i(m)$ allow us to bypass the calculation of the targeting distribution $P_i(m|M)$, some useful statistical information can still be obtained. For example:

- $\bar{m}_i \equiv$ expected number of missiles targeting ship i given that it is present

$$= MP_i(1|1) \ . \tag{14}$$

Calculation of $P_i(1|1) \equiv \bar{p}_i$ is particularly simple (reference 1).

- The expected number of missile hits on ship i can also be calculated without knowing $P_i(m|M)$. If there are no point defenses, it is simply $\bar{m}_i$. If there are point defenses (which would be included in the lethality functions), it is obtained by matching the probability of sink P_{Ki} with the curve giving probability of sink versus number of hits received.

PROOFS

Proof of Targeting Distribution Formulas

To prove the sum result in equation (1), consider a subset of k ships containing the ith ship. The ships in the subset are labelled $i, j_2, \ldots, j_k$. The probability that this subset is realized is $a_i a_{j_2} \ldots a_{j_k} (1-a_{j_{k+1}}) \ldots (1-a_{j_n})$, and the probability that a given missile will target the ith ship from this subset is $p_i = w_i/(w_i+w_{j_2}+\ldots+w_{j_k})$. Since the missiles are independent, the distribution for m missiles impacting is binomial with probability p_i. One must then sum over all possible ways of forming a subset of k ships containing the ith ship, and sum also over all possible sizes of the subset (k varying from 1 to n). Each of the 2^{n-1} possible configurations of ships -- from the ith ship alone to all ships present -- will then have been included.

Let us now turn to the integral representation (3). The case for 1 missile (m=M=1) was treated in reference 1, and the procedure is generalized here. We can establish the following lemma.

Lemma

The following sum:

$$\sum_{k=1}^{n} \sum_{\{j_2,\ldots,j_k\}=\{1,\ldots,n;\text{except } i\}} a_{j_2}\ldots a_{j_k}(1-a_{j_{k+1}})\ldots(1-a_{j_n})\psi(p_i) \tag{15}$$

where ψ is an arbitrary function and $p_i \equiv w_i/(w_i+w_{j_2}+\ldots+w_{j_k})$, is equal to the following integral:

$$w_i \int_0^1 dx\, F_i(x) x^{w_i-1} \prod_{j=1;\neq i}^{n} (a_j x^{w_j}+1-a_j), \tag{16}$$

where $F_i(x)$ is the inverse Laplace transform of $\psi(1/s)$ evaluated at $t = -w_i \text{Log } x$, when such a transform exists.[1] [The Laplace transform of $q(t)$ is $\int_0^\infty e^{-st}q(t)dt$].

[1] $F_i(x)$ can be a distribution, rather than a function defined at every point. All that is required is that the integral in (16) be defined. See below for an example where F is the distribution called the delta "function".

Proof of Lemma. Both the integral and the sum have 2^{n-1} terms. A typical term in the integrand of (16) is

$$w_i a_{j_2} \cdots a_{j_k} (1-a_{j_{k+1}}) \cdots (1-a_{j_n})\, x^{w_i+w_{j_2}+\cdots+w_{j_k}-1} F_i(x) \quad .$$

To match this with the typical term in the sum (15), we must have

$$w_i \int_0^1 x^{w_i+w_{j_2}+\cdots+w_{j_k}-1} F_i(x)dx = \psi\left(\frac{w_i}{w_i+w_{j_2}+\cdots+w_{j_k}}\right) \quad , \tag{17}$$

If we make the change of variable $x^{w_i} = y$, then define $s \equiv 1 + (w_{j_2}+\cdots+w_{j_k})/w_i$ and $\phi_i(y) = F_i(y^{1/w_i})$, the problem reduces to finding a function ϕ_i independent of s such that

$$\int_0^1 y^{s-1} \phi_i(y)dy = \psi(\tfrac{1}{s}) \quad .$$

With the change of variable $y=e^{-t}$, this becomes:

$$\int_0^\infty e^{-st} \phi_i(e^{-t})dt = \psi(\tfrac{1}{s}) \quad .$$

The left-hand side is the definition of the Laplace transform of $\phi_i(e^{-t})$. Substituting back $t = -\text{Log } y$ and $y = x^{w_i}$ gives the result stated in the lemma.

In this case, we have

$$\psi(p_i) = p_i^m (1-p_i)^{M-m} \quad .$$

so

$$\psi(\frac{1}{s}) = \frac{(s-1)^{M-m}}{s^M} = \sum_{\ell=0}^{M-m} \binom{M-m}{\ell}(-1)^{\ell}/s^{m+\ell} \quad . \tag{18}$$

Taking the inverse Laplace transform of the right-hand side gives

$$\phi_i(e^{-t}) = \sum_{\ell=0}^{M-m} \binom{M-m}{\ell}(-1)^{\ell}\, t^{\ell+m-1}/(\ell+m-1)! \tag{19}$$

for $m\neq 0$. Substituting back $t = -\text{Log } y$ and $y = x^{w_i}$ gives the $g_i(x)$ in equation (5). When $m=0$, the $\ell=0$ term in (19) is undefined. Equation (18) shows that this term is the inverse Laplace transform of 1, which is the delta function[1] $\delta(t)$. Thus,

$$\phi_i(e^{-t}) = \delta(t) + \sum_{\ell=1}^{M} \binom{M}{\ell}(-1)^{\ell}\, t^{\ell-1}/(\ell-1)! \tag{20}$$

for $m=0$. Substituting back to y, then x, and putting the resulting $g_i(x)$ in (4) yields the expression (6) for $P_i(0|M)$. A change of variable $-\text{Log } x = u$ and the fact that $\int_a^b \delta(x)f(x)dx = f(0)$ if $0\in[a,b]$ must be used.

Note that g_i is essentially a Laguerre polynomial. Indeed, the Laplace transform of $t^{M-1}(M-m)!\, L_{M-m}^{m-1}(t)/(M-1)!$ is $(s-1)^{M-m}/s^M$, which is the right-hand side of (18) (reference 2, p. 191). Thus, for $m\neq 0$,

$$g_i(x) = (-w_i \text{Log } x)^{M-1}\,(M-m)!\; L_{M-m}^{m-1}(-w_i \text{ Log } x)/(M-1)! \tag{21}$$

---

[1] The step function $\vartheta(t-k)$, which is 1 for $t>k$ and 0 otherwise, has the Laplace transform e^{-ks}/s. Its derivative with respect to k, $-\delta(t-k)$, has the Laplace transform $-e^{-ks}$. As $k\to 0^+$, $\delta(t)$ has the Laplace transform 1.

where L is the Laguerre polynomial, $L_n^a(x) \equiv \sum_{m=0}^{n} \binom{n+a}{n-m}(-x)^m/m!$

Proof of Survival Probabilities Formula

For a lethality function $L_i(m)$ given by (9), one can readily establish that $P_{Ki}(M)$ is given by (10) by using the following results:

$$\sum_{m=0}^{M} \binom{M}{m} p_i^m (1-p_i)^{M-m} = 1 \tag{22}$$

$$\sum_{m=0}^{M} \binom{M}{m} \alpha^m p_i^m (1-p_i)^{M-m} = (\alpha p_i + 1 - p_i)^M \tag{23}$$

$$\sum_{m=0}^{M} \binom{M}{m} m\alpha^m p_i^m (1-p_i)^{M-m} = M\alpha p_i(\alpha p_i + 1 - p_i)^{M-1} \tag{24}$$

Equations (22) and (23) are simple binomial expansions of $(a+b)^n$. Equation (24) can be obtained from (23) by differentiating with respect to α then multiplying by α.

To find the $u_i(x)$ in the integral form, we proceed as before. With reference to the lemma established earlier, we have:

$$\psi(p_i) = 1 - (1-d_ip_i)^M + Mc_ip_i(1-b_i)(1-b_ip_i)^{M-1} \tag{25}$$

so

$$\psi\left(\frac{1}{s}\right) = 1 - \left(\frac{s-d_i}{s}\right)^M + Mc_i(1-b_i)\frac{(s-b_i)^{M-1}}{s^M}$$

$$= -\sum_{\ell=1}^{M} \binom{M}{\ell} \frac{(-d_i)^\ell}{s^\ell} + Mc_i(1-b_i)\sum_{\ell=0}^{M-1} \binom{M-1}{\ell} \frac{(-b_i)^\ell}{s^{\ell+1}} \tag{26}$$

Taking inverse Laplace transforms as before gives $\phi_i(e^{-t})$, and hence $u_i(x)$. A change of index in the first sum then puts it in the form shown in (12).

Formula (14) is proved by observing that

$$\sum_{m=0}^{M} m\binom{M}{m}p_i^m(1-p_i)^{M-m} = Mp_i \quad \text{(i.e., (24) with } \alpha=1\text{)} .$$

PROPERTIES OF THE INTEGRAL REPRESENTATIONS

The integrand $f_i(x)$ in (3) has the following properties:

(1) $f_i(0) = 0$ if $w_i>1$ because of the presence of x^{w_i-1} and the fact that for any positive integer N, $[(\text{Log } x)^N x]$ tends to 0 when x tends to 0. If $w_i=1$, then $f_i(0)$ can be infinite. Since this should be avoided in numerical integration, it is preferable to take all weights w_i larger than 1, which does not change the end result.

(2) $f_i(1) = \begin{cases} 0 & \text{if } m>1 \\ 1 & \text{if } m=1 \end{cases}$

(3) The polynomial in x entering the expression of $f_i(x)$ has positive coefficients, and hence no roots for x in [0,1]. The function $g_i(x)$, which is a polynomial in the positive quantity $y= -\text{Log } x$ whose coefficients have alternating signs, has M-m positive roots y_r (reference 2, p. 204). Therefore, $f_i(x)$ has M-m roots ($x_r = e^{-y_r}$) between 0 and 1, excluding 0 and 1. Thus, the graph of $f_i(x)$ is oscillatory with (M-m)/2 cycles. The graphs of $h_i(x)$ in (6) and $u_i(x)$ in (12) are also oscillatory.

(4) The case $M=m=1$ is particularly simple to integrate numerically and was treated in reference 1. $f(x)$ is a polynomial with positive coefficients, and hence its graph is concave upwards. The maximum error using the trapezoidal rule or better was given as $[2f(0) + f(1) - f(1-1/N)]/2N$, where N is the number of (equal) intervals used. A refinement[1], due to David A. Perin, gives a maximum error of only

$$E = [f(0) + f(1) - f(1-1/4N) - f(1/4N)]/2N \ . \qquad (27)$$

Verifications

Any program which calculates $P_i(m|M)$ should include the following simple checks:

$$\bullet \sum_{m=0}^{M} P_i(m|M) = 1 \qquad (28)$$

$$\bullet \sum_{m=0}^{M} m\, P_i(m|M) = M\, P_i(1|1) \qquad (29)$$

The quantity $P_i(1|1) \equiv \bar{p}_i$ must be calculated separately as described earlier.

---

[1] Instead of extending the chord from x_i to x_{i+1} to obtain a lower bound on the next interval, as was done in reference 1, consider the tangent to the curve at each interval point x_i (except the endpoints 0 and 1), and extend it until it crosses the mid-interval lines on either side [$y = (x_i + x_{i-1})/2$ and $y = (x_i + x_{i+1})/2$] . The area of the resulting right trapezoid is simply $f(x_i)/N$. This gives a lower bound for all but 2 half-intervals, one at each endpoint. Drawing the tangent at the mid-point of each half-interval [at $x_1/4$ and $(3 + x_{N-1})/4$] gives the total lower bound:

$$\int_0^1 f(x)dx > \frac{1}{N}[f(\tfrac{1}{N}) + \ldots + f(1-\tfrac{1}{N})] + \frac{1}{2N}\,[f(\tfrac{1}{4N}) + f(1-\tfrac{1}{4n})] \ .$$

EXPECTED-VALUE APPROACHES

Expected-value constructions of the targeting distribution $P_i(m|M)$ consist in calculating an average probability $\bar{p}_i$ that a single missile will target ship i given that ship i is present and then constructing a binomial distribution based on this average probability. Thus

$$P_i(m|M) = \binom{M}{m} \bar{p}_i^m (1-\bar{p}_i)^{M-m} \quad . \tag{30}$$

Some possibilities for $\bar{p}_i$ are:

Expected-Value Approach No. 1: (Semi-Exact)

$$\bar{p}_i = P_i(1|1) = \sum_{k=1}^{n} \sum_{\{j_2,\ldots,j_k\} = \{1,\ldots,n;\text{except } i\}} a_{j_2}\cdots a_{j_k}$$

$$\cdot (1-a_{j_{k+1}})\cdots(1-a_{j_n}) \frac{w_i}{w_i+w_{j_2}+\ldots+w_{j_k}}$$

$$= w_i \int_0^1 x^{w_i-1} \prod_{j=1,\neq i}^{n} (a_j x^{w_j}+1-a_j)dx \quad . \tag{31}$$

Expected-Value Approach No. 2:

$$\bar{p}_i = \frac{a_i w_i}{\sum_{j=1}^{n} a_j w_j} \quad . \tag{32}$$

Expected-Value Approach No. 3:

$$\bar{p}_i = \frac{w_i}{\sum_{j=1}^{n} a_j w_j} \left[1 - \prod_{s=1}^{n}(1-a_s)\right] . \tag{33}$$

Expected-Value Approach No. 4:

$$\bar{p}_i = \frac{w_i}{w_i + \sum_{j=1,\neq i}^{n} a_j w_j} . \tag{34}$$

Expected-Value Approach No. 5:

$$\bar{p}_i = \frac{k w_i}{w_i + \sum_{j=1,\neq i}^{n} a_j w_j} \quad ; \quad k \equiv \frac{1 - \prod_{s=1}^{n}(1-a_s)}{\sum_{i=1}^{n}\left(\frac{a_i w_i}{w_i + \sum_{j=1;\neq i}^{n} a_j w_j}\right)} . \tag{35}$$

Expected-Value Approach No. 6:

$$\bar{p}_i = \frac{w_i}{\sum_{j=1}^{n} w_j} \qquad (\text{all } a_j = 1) . \tag{36}$$

No. 1 is referred to as "semi-exact" because the $\bar{p}_i$ it uses is exact. No. 2 takes the ratio of the expected weight of ship i to the total expected weight of the formation. No. 3 takes the full weight of the ith ship (since $\bar{p}_i$ assumes ship i is present) divided by the expected weight, and normalizes the result, so that the probability of targeting something, namely $\sum_{i=1}^{n} a_i \bar{p}_i$, is equal to the probability that something is present, namely $1 - \prod_s (1-a_s)$. Its drawback is that $\bar{p}_i$ can be larger than 1 (if the a_js are small enough). No. 4 argues that since the ith ship is assumed present in $\bar{p}_i$, it should contribute its full weight when the expected weight of the formation is computed in the denominator. No. 5 normalizes No. 4. No. 6 assumes that all ships are present for purposes of calculating the conditional probability, with a view of correcting this assumption when all $P_i(m|M)$ are multiplied by a_i at the end to yield the unconditional probability. All approaches, exact or expected-value, must reduce to No. 6 when all a_js are indeed 1. No. 6 gauges the sensitivity of the resulting probability of targeting ship i given that it is present to whether or not ship i is, in fact, present.

Critique of the Expected-Value Approaches: Examples

We will compare the various approaches in 2 examples. Both include $n = 11$ ships. In the first the formation is attacked with $M = 12$ missiles. In the second the weights are slightly changed and $M = 40$ missiles attack. The data are:

Example 1:

a_i = .379, .249, .605, .553, .458, .695, .258, .695, .437, .460, .553

w_i = 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1

M = 12

Example 2:

a_i = .379, .249, .605, .553, .458, .695, .258, .695, .437, .460, .553

w_i = 4, 4, 1, 1, 1, 1, 1, 1, 1, 1, 1

M = 40

The exact and the 6 expected-value targeting distributions for ship 1, $P_1(m|M)$, are shown in figures 1 and 2[1]. We can make the following observations:

- When only 12 missiles are launched, many expected-value approaches come reasonably close to the correct result. When as many as 40 missiles are launched, all expected-value approaches break down. They all lead to narrow, sharply-peaked distributions whereas the correct distribution is noticeably spread out.

[1]The $\bar{p}_1$ on which the binomial distributions are based are:

Expected-value method number →	1	2	3	4	5	6
$\bar{p}_1$ (example 1)	.295	.127	.335	.277	.306	.154
$\bar{p}_1$ (example 2)	.435	.210	.553	.412	.474	.235

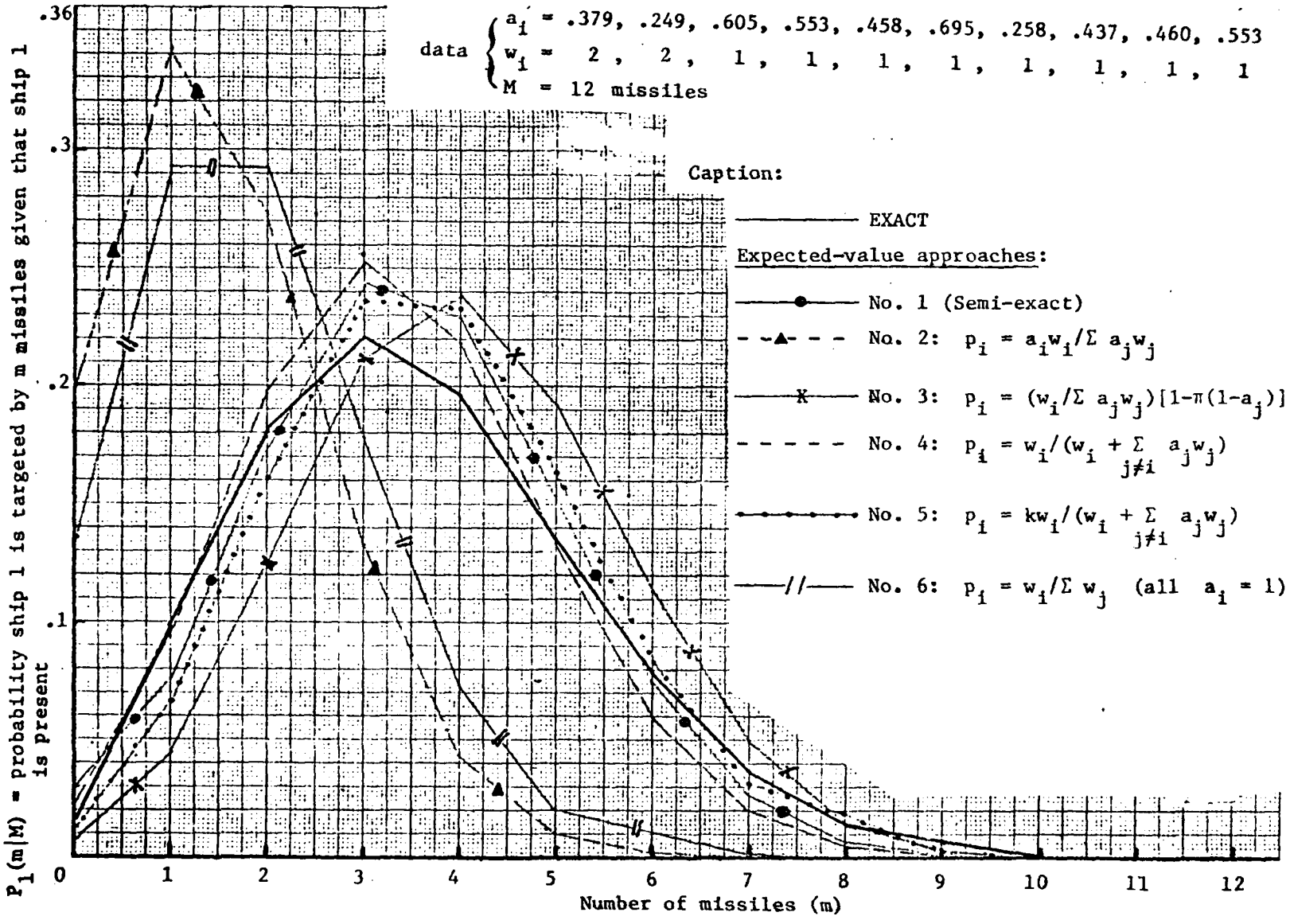

FIG. 1: Targeting Distribution in Example 1

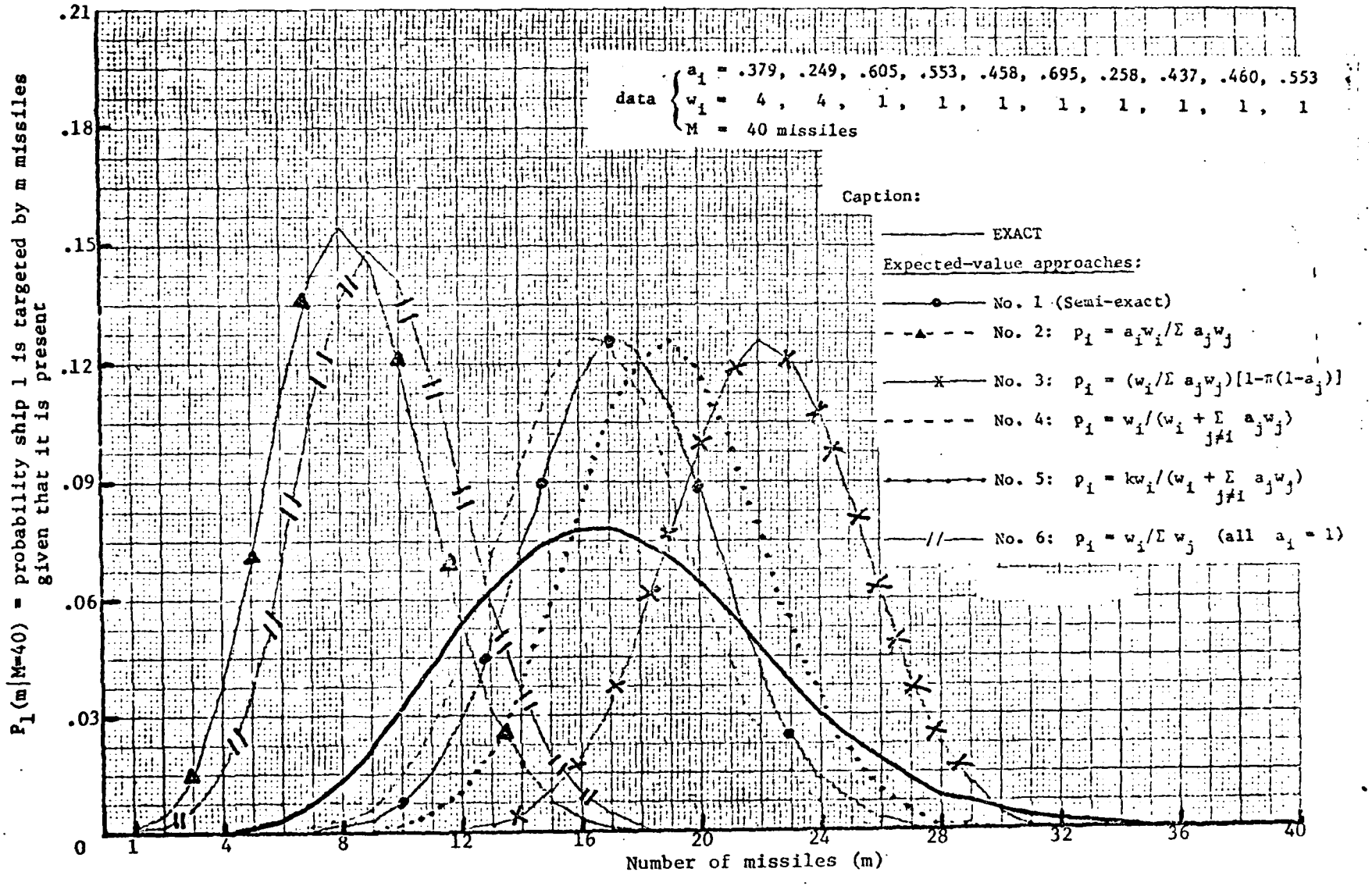

FIG. 2: Targeting Distribution in Example 2

- Approaches Nos. 2 and 6 lead very far from the correct result in both cases.
- The semi-exact result (No. 1) is closest to the exact result in both cases.
- No. 3 peaks later than all the others.
- Nos. 1, 4, and 5 are close to one another in both cases.

The wider spread of the exact distribution can be predicted by calculating variances. For example, the variance of the exact distribution (calculated by James K. Tyson) is:

$$\sigma_m^2 = M\,\bar{p}_i(1-\bar{p}_i) + M(M-1)\sigma_p^2 \quad ,$$

where $\sigma_p^2 \equiv \overline{p_i^2} - \bar{p}_i^2$ is independent of M, but the variance of the semi-exact distribution, which is a single binomial, is

$$\sigma_m^2 = M\,\bar{p}_i(1-\bar{p}_i) \quad .$$

Thus, the exact distribution is always more spread out than the semi-exact expected-value distribution, and the difference between the 2 standard deviations grows as M, the total number of missiles. For a very large number of missiles, one would witness a spectacular breakdown of all the expected-value approaches considered here.

Let us now introduce the lethality (Equation (8)) to find the probability P_{K1} that ship 1 is sunk given it is present. Two lethality functions are considered: $L_1(m) = 1 - (1 - .01)^m$ and $L_1(m) = 1 - (1 - .03)^m$. The results for example 2 are given in the table below.

Lethality function $L_1(m)$	P_{K1}						
	Exact	No. 1 (semi-exact)	No. 2	No. 3	No. 4	No. 5	No. 6
$1 - (1-.01)^m$	.159	.160	.081	.199	.152	.173	.090
$1 - (1-.03)^m$	.404	.408	.223	.488	.392	.436	.247

As can be seen, the semi-exact result is extremely close to the exact result, and Nos. 4 and 5 are not far behind. The other expected-value methods break down. It appears therefore, that, in this particular case, expected-value distributions having the same mean as the exact one yield results close to the exact one for power-law lethality functions. However, this is not true for other realistic lethality functions. For example, consider the case of the cookie-cutter lethality function, $L_i(m) = 0$ for $m \leq m_o$ and 1 for $m > m_o$, corresponding to a saturation effect with nuclear missiles (the point defenses destroy any number of missiles smaller than m_o, but if more than m_o engage, the excess will hit the ship and sink it with certainty). The sink probability is then simply $P_{Ki}(M) = \sum_{m=m_o}^{M} P_i(m|M)$, which can vary widely depending on m_o and the approach used, as figures 1 and 2 testify.

The reason why the sink probabilities given by some expected-value results are close the exact sink probability in this case is that in the interval where most of the distribution is concentrated (m between 12 to 24) the lethality function does not vary dramatically, so that the shape of the distribution is not a dominant consideration.

Conclusion on Expected-Value Approaches

The conclusions to be drawn from the above are:

- When the object is to calculate the targeting distribution, all expected-value approaches fail significantly for large numbers of missiles (~40), but some are successful for small numbers of missiles (~12).

- When the object is to calculate the probability of sinking a ship, all expected-value approaches fail significantly for some lethality functions (such as a cookie-cutter), but some are successful for other lethality functions (such as power laws).
- The semi-exact expected-value approach (No. 1) yields results closest to the exact approach. Approaches Nos. 4 and 5 are not far behind. The other approaches fail significantly in all cases.

As can be seen, results yielded by reasonable expected-value approaches can show large errors when compared to results yielded by the corresponding exact approach. This is all the more sobering as in many cases the problem is so complex (e.g., a full campaign) that an exact approach is almost impossible to formulate, let alone to use for calculations.

More details and computer programs can be found in reference 3.

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